Appendix B

Mathematics Teacher TPACK Development Model: Themes X Levels X Descriptors X Examples

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### CURRICULUM & ASSESSMENT

<table>
<thead>
<tr>
<th>C: Curriculum descriptor</th>
<th>A: Assessment descriptor</th>
<th>Ex: Mathematics Example</th>
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#### Recognizing

- **C**: Acknowledges that mathematical ideas displayed with the technologies can be useful for making sense of topics addressed in the curriculum.  
  Ex: Creates graphs of multiple linear functions using graphing calculators to provide a visual representation for varying slopes. Considers these visuals as making sense of the idea of slope but is unsure of how this might help students learn the basic concept.
- **A**: Resists idea of technology use in assessment indicating that technology interferes with determining students’ understanding of mathematics.  
  Ex: Does not allow calculator use when assessing students’ understanding of solving linear equations.

#### Accepting

- **C**: Expresses desire but demonstrates difficulty in identifying topics in own curriculum for including technology as a tool for learning.  
  Ex: Attends and participates in mathematics dynamic geometry system workshop to identify curricular ideas for incorporating the technologies as learning tools. Mimics the incorporation of a dynamic geometry system idea from the workshop to display measuring the sum of the angles of a triangle that upon multiple changes of the triangle suggests that the sum of the angles of any triangle is 180 degrees.
- **A**: Acknowledges that it might be appropriate to allow technology use as part of assessment but has a limited view of its use (i.e., use of technology on a section of an exam).  
  Ex: Attends and participates in a mathematics assessment professional development to consider ideas for assessing students’ understanding of solving systems of linear functions using the calculator as a tool. Mimics the assessment idea to explain the use of the calculator for solving systems of linear functions by using the trace function to identify the intersection. Often retests technology questions with paper and pencil questions to be sure that the concept was learned the ‘right’ way.

#### Adapting

- **C**: Understands some benefits of incorporating appropriate technologies as tools for teaching and learning the mathematics curriculum.  
  Ex: Targets key topics students investigate with technology. Develops lessons to demonstrate mathematics concepts with technology and activities for students to use technology to verify or reinforce those concepts. After students have learned to create graphs of specific linear functions, students are challenged to use the spreadsheet to verify the graphical representation of the ordered pairs.
- **A**: Understands that if technology is allowed during assessments that different questions/items must be posed (i.e., conceptual vs. procedural understandings).  
  Ex: Allows use of calculator in an assessment but designs the assessment to focus on gathering students’ conceptual understanding of solving systems of linear functions in addition to their procedural understanding.

#### Exploring

- **C**: Investigates the use of topics in own curriculum for including technology as a tool for learning; seeks ideas and strategies for implementing technology in a more integral role for the development of the mathematics that students are learning.  
  Ex: Adapts own previous mathematics lesson to include technology.  
  Ex: Develops own ideas about using technology to enhance current curriculum; thus, begins altering preexisting activities or creating new activities for current curriculum.
### Actively investigates use of different types of technology-based assessment items and questions (e.g., technology active, inactive, neutral or passive).
- Ex: Designs assessments where students are expected to show their understanding of mathematical ideas using an appropriate technology that extends beyond paper and pencil type questions.

### Advancing C:
- Understands that sustained innovation in modifying own curriculum to efficiently and effectively incorporate technology as a teaching and learning tool is essential.
- Ex: Develops innovative ways to use technology to develop mathematical thinking in students such as using virtual algebra tiles to extend ideas of handheld manipulatives to focus on variables in algebraic expressions.
- Ex: Modifies and advances curriculum to take advantage of technology as a tool for teaching and learning such as using CAS to explore more complex algebraic expressions.

### Actively reflects on and adapts assessment practices that examine students’ conceptual understandings of the subject matter in ways that demand full use of technology.
- Ex: Develops innovative assessments to capture students’ understandings of the mathematics embedded in the particular technology.

### LEARNING

<table>
<thead>
<tr>
<th>M: Mathematics learning descriptor</th>
<th>C: Conception of student thinking descriptor</th>
<th>Ex: Mathematics example</th>
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<tbody>
<tr>
<td>Recognizing M: Views mathematics as being learned in specific ways and that technology often gets in the way of learning.</td>
<td>More apt to accept the technology as a teaching tool rather than a learning tool.</td>
<td>Technology is used only outside of normal classroom activities, such as checking homework, calculating large numbers, etc.</td>
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<tr>
<td>Accepting M: Has concerns about students’ attention being diverted from learning of appropriate mathematics to a focus on the technology in the activities.</td>
<td>Is concerned that students do not develop appropriate mathematical thinking skills when the technology is used as a verification tool for exploring the mathematics.</td>
<td>Activities that use technology are almost always redone without technology to be certain students really learned the particular concept.</td>
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<td>Adapting M: Begins to explore, experiment and practice integrating technologies as mathematics learning tools.</td>
<td>Begins developing appropriate mathematical thinking skills when technology is used as a tool for learning.</td>
<td>Although students use technology for most topics, assessing student thinking remains mostly technology free.</td>
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<tr>
<td>Exploring M: Uses technologies as tools to facilitate the learning of specific topics in the mathematics curriculum.</td>
<td>Plans, implements, and reflects on teaching and learning with concern for guiding students in understanding.</td>
<td>Technology activities are implemented and evaluated with respect to student learning of mathematics and student attitudes toward mathematics.</td>
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<tr>
<td>Advancing</td>
<td>Manages technology-enhanced activities towards directing student engagement and self-direction in learning mathematics.</td>
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M: Plans, implements, and reflects on teaching and learning with concern and personal conviction for student thinking and understanding of the mathematics to be enhanced through integration of the various technologies.
Ex: Students explore mathematics topics, integrating various technologies in attempts to better understand mathematical concepts.

C: Technology-integration is integral (rather than in addition) to development of the mathematics students are learning.
Ex: Engages students in high-level thinking activities (such as project-based and problem solving and decision making activities) for learning mathematics using the technology as a learning tool.
Ex: Technology is used to develop advanced levels of understanding of mathematical concepts.

**TEACHING**

M: Mathematics learning descriptor  I: Instructional descriptor E: Environment descriptor PD: Professional development descriptor Ex: Mathematics example

**Recognizing**
M: Concerned that the need to teach about the technology will take away time from teaching mathematics.
Ex: Students use technology on their own and little or no instruction with technology is present.
I: Does not use technology to develop mathematical concepts.
Ex: Technology, if used in class, is used for menial or rote activities.
E: Uses technology to reinforce concepts taught without technology.
Ex: Focus on linear functions where students practice creating graphs by hand to explore different functions. After students have demonstrated competence with linear functions, summarize the knowledge, with a spreadsheet example or a graphing calculator example.
PD: Considers attending local professional development to learn more about technologies.
Ex: Attends local workshops that focus on gaining skills with the technology; context of the learning activities is mathematics.

**Accepting**
M: Uses technology activities at the end of units, for “days off,” or for activities peripheral to classroom instruction.
Ex: Technology-enhanced activities are not used for topics that require more advanced technology skills.
I: Merely mimics the simplest professional development mathematics curricular ideas for incorporating the technologies.
Ex: Introduces the Pythagorean Theorem algorithmically; teacher use of dynamic geometry to verify the Pythagorean Theorem; students find solutions to example problems using paper and pencil.
E: Tightly manages and orchestrates instruction using technology.
Ex: Technology is directed, in a tightly sequenced, step-by-step process. Skill-based, non-exploratory technology use.
PD: Recognizes the need to participate in technology related PD.
Ex: Seeks out technology-related professional development, workshops that are directed at developing the technology in the learning of mathematics.

**Adapting**
M: Uses technology to enhance or reinforce mathematics ideas that students have learned previously.
Ex: Students use technology to reinforce previously teacher-taught concepts.
I: Mimics the simplest professional development activities with the technologies but attempts to adapt lessons for his/her mathematics classes.
Ex: Technology-based lessons are incorporated that are tailored to students’ needs.
E: Instructional strategies with technologies are primarily deductive, teacher-directed in order to maintain control of the how the activity progresses.
Ex: Begins to adapt instructional approaches that allow students opportunities to explore with technology for part of lessons.
PD: Continues to learn and explore ideas for teaching and learning mathematics using only one type of technology (such as spreadsheets).
Ex: Shares ideas from professional development with other mathematics teachers in the building.

**Exploring**

**M:** Engages students in high-level thinking activities (such as project-based and problem solving and decision making activities) for learning mathematics using the technology as a learning tool.

   Ex: Teachers share classroom-tested, technology-based lessons, ideas, and successes with peers.

**I:** Engages students in explorations of mathematics with technology where the teacher is in role of guide rather than director of the exploration.

   Ex: Students use technology to explore new concepts as the teacher serves mostly as a guide.

**E:** Explores various instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics.

   Ex: The teacher incorporates a variety of technologies for numerous topics.

**PD:** Seeks out and works with others who are engaged in incorporating technology in mathematics.

   Ex: Organizes teachers of similar mathematics and grade level in investigating the mathematics curriculum to integrate appropriate technologies.

**Advancing**

**M:** Active, consistent acceptance of technologies as tools for learning and teaching mathematics in ways that accurately translate mathematical concepts and processes into forms understandable by students.

   Ex: Teacher is seen as a resource as novel ideas for helping students learn mathematics with technology.

**I:** Adapts from a breadth of instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics.

   Ex: The teacher helps students move fluently from one tool to another while demonstrating a focus on and a joy of deeply understanding mathematical topics.

**E:** Manages technology-enhanced activities in ways that maintains student engagement and self-direction in learning the mathematics.

   Ex: The teacher forms and reforms learning groups where individual and group learning is valued and encouraged.

**PD:** Seeks ongoing PD to continue to learn to incorporate emerging technologies. Continues to learn and explore ideas for teaching and learning mathematics with multiple technologies to enhance access to mathematics.

   Ex: Engages teachers in the district in evaluating and revising the mathematics curriculum to more seamlessly integrate technology throughout the grades, adjusting the curriculum for a 21st century mathematics curriculum with appropriate technologies.

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**ACCESS**

**U:** Usage descriptor  **B:** Barrier descriptor  **A:** Availability descriptor  **Ex:** Mathematics example

**Recognizing**

**U:** Permits students to use technology ‘only’ after mastering certain concepts.

   Ex: Mathematical exploration with technology tools is challenged by beliefs about how students need to learn mathematics.

**B:** Resists consideration of changes in content taught although it becomes accessible to more students through technology.

   Ex: Student access to technology is limited to ‘after’ they have learned the given concepts using paper and pencil procedures and only for rote activities.

**A:** Notices that authentic problems are more likely to involve ‘unfriendly numbers’ and may be more easily solved if students had calculators.

   Ex: Assigns some mathematics problems using school and community data but saves then for “extra credit” work if students have calculators.

**Accepting**
U: Students use technology in limited ways during regular instructional periods.
   Ex: Student activities with technology are limited to brief tightly controlled situations.

B: Worries about access and management issues with respect to incorporating technology in the classroom.
   Ex: Students can only use technology in isolated situations or non-important learning situations.

A: Calculators permit greater number of examples to be explored by students.
   Ex: Student use calculators to investigate patterns and functions.

Adapting

U: Permits students to use technology in specifically designed units.
   Ex: Access to and use of technology is available for exploration of new topics, usually with the teacher’s demonstration.

B: Uses technology as a tool to enhance mathematics lessons in order to provide students a new way to approach mathematics.
   Ex: Concepts learned with technology are not assessed with technology.

A: Concepts are taught differently since technology provides access to connections formerly out of reach.
   Ex: Students use dynamic geometry software to investigate and make connections between trigonometry functions.

Exploring

U: Permits students to use technology for exploring specific mathematical topics.
   Ex: Access to and use of technology is available and encouraged for mathematics exploration during most class times.

B: Recognizes challenges for teaching mathematics with technologies, but explores strategies and ideas to minimize the impact of those challenges.
   Ex: Technology is used extensively in assessments. Seeks out ways to obtain technology for classroom use and begins creating methods for technology management issues.

A: Through the use of technology, key topics are explored, applied, and assessed incorporating multiple representations of the concepts and their connections.
   Ex: Simultaneous equations are developed from an authentic situation, solved, and interpreted using graphs, tables, symbols and data.

Advancing

U: Permit students to use technology in every aspect of mathematics class.
   Ex: Technology is seen as an opportunity to challenge notions of what mathematics students can master.

B: Recognizes challenges in teaching with technology and resolves the challenges through extended planning and preparation for maximizing the use of available resources and tools.
   Ex: Technology is used to expand the mathematics concepts that can be accessed by students.

A: Students are taught and permitted to explore more complex mathematics topics or mathematical connections as part of their normal learning experience.
   Ex: Using the Internet to find interesting mathematical problems, students investigate the role that technologies can play in finding solutions to the problems.